Real-Time, Automatic Vehicle-Potential Determination from ESA Measurements: The Distribution Function Algorithm

Stanley L. Spiegel*
University of Lowell, Lowell, Massachusetts
and
Herbert A. Cohen†
Air Force Geophysics Laboratory, Hanscom Air Force Base, Massachusetts

A technique has been developed that uses electrostatic analyzer (ESA) positive ion count measurements at geosynchronous orbit to compute the plasma distribution function and, from this, to infer space vehicle potential in real time with high accuracy.

I. Introduction

THE development and testing of a computer algorithm, called the "count-ratio" algorithm, which uses electrostatic analyzer (ESA) ion count data for the real-time determination of satellite frame potential, has been described in a previous paper[1] hereafter referred to as "CR." This algorithm examines ion counts in adjacent ESA energy channels and makes a determination that the vehicle is charged if the upper-to-lower energy channel count ratio exceeds an empirically determined critical value, with a further requirement that, for statistical reliability, some specified minimum number of counts be recorded in the upper channel. As reported in CR, the count-ratio algorithm was found to perform very well when tested on a data base consisting of 9925 positive ion count spectra recorded at geosynchronous orbit during time periods on 30 different days by the high resolution (64 channel) ESA of the SC9 experiment onboard the P78-2 satellite (see Stevens and Vampola;[2] the SC9 experiment was designed, built, and operated by the University of California at San Diego).

Encouraged by these results, but troubled by the algorithm's reliance on, and sensitivity to, two ad hoc parameters (critical count ratio, minimum count number), we were motivated to develop a related method that bases its determination of charging on examination of the ion distribution function, a physical property of the plasma readily computed from the ion count spectrum, rather than directly from the ion counts themselves (whose values, and thus ratio, depend on ESA design as well as on the ambient plasma). The resultant "distribution function" algorithm also exploits the jump in ion count at the energy of charging, but in a more physically meaningful way: a determination of charging is made only if the jump in counts implies a statistically significant increase in the computed ion distribution function (see Sec. II). There is no need to specify the two instrument-dependent parameters, count minimum and critical ratio, with this algorithm. In tests with the 9925 spectra data base referred to above, the distribution function algorithm has been found to give excellent results.

Received Oct. 23, 1986; revision received Aug. 8, 1987.

*Associate Professor, Mathematics Department.

†Physicist, Space Physics Division, presently Senior Systems Engineer, Schafer Associates, Arlington, VA. Member AIAA.

II. Description of the Distribution Function Algorithm

Given an observed ion count spectrum (as measured by the ESA of the SC9 experiment) one can compute the positive ion distribution function. Its average value for protons over the energy interval $\Delta E(I)$ associated with the I-th ESA channel ($I = 0,1,\ldots,63$), with midchannel energy $E(I)$, is given by

$$F(I) = \frac{5.45 \times 10^5 C(I)}{E(I)\Delta E(I)G(I)} \tag{1}$$

(see, for example, Mullen et al.[3]). Here $C(I)$ is the count rate observed in the I-th channel, and $G(I)$ is the so-called geometric factor of the ESA, a function of instrument design and efficiency which is determined by careful calibration for each channel. The numerical factor is chosen so that if $C(I)$ is given in counts per second, $(E)I$ and $\Delta E(I)$ in eV, and $G(I)$ in cm^2-sr, $F(I)$ will be in its customary units of s^3/km^6.

Now it is known that on physical grounds for a stable plasma environment, the distribution function is a decreasing function of energy, regardless of plasma temperature (or temperatures if more than one population is present). However, if the spacecraft is at a negative potential with respect to plasma ground, the ion energy spectrum will be shifted by an amount equal to the vehicle potential. Therefore, a statistically significant increase in the computed ion distribution function at a given ESA channel serves as an indicator of spectrum shift, thus implying vehicle charge to the corresponding energy level (see de Forest;[4] this phenomenon corresponds, of course, to the jump in ion count rate searched for by the count rate algorithm, described in CR).

(While this test for charging is predicted on a stable plasma environment, the distribution function algorithm which incorporates it has been found to work extremely well in tests with ion count data from a wide range of spectra for which the presence or absence of plasma stability is not known; results of these algorithm tests are given in Sec. III below.)

Since the distribution function is derived from a randomly fluctuating count rate, it is necessary to ensure that any computed increase in distribution function be statistically significant. Thus the criterion for determining that the vehicle is charged is that $F(I+1) - F(I)$ exceed zero by some number of standard deviations of the distribution function difference. To compute this, one must first compute the standard deviation in $F(I)$.

Using the energy dependencies of $G(I)$ and $\Delta E(I)$ valid for the SC9 ESA which are given in Appendix A.4 of Mullen et al.,[3] it is possible to rewrite the expression for $F(I)$ in Eq. (1) as

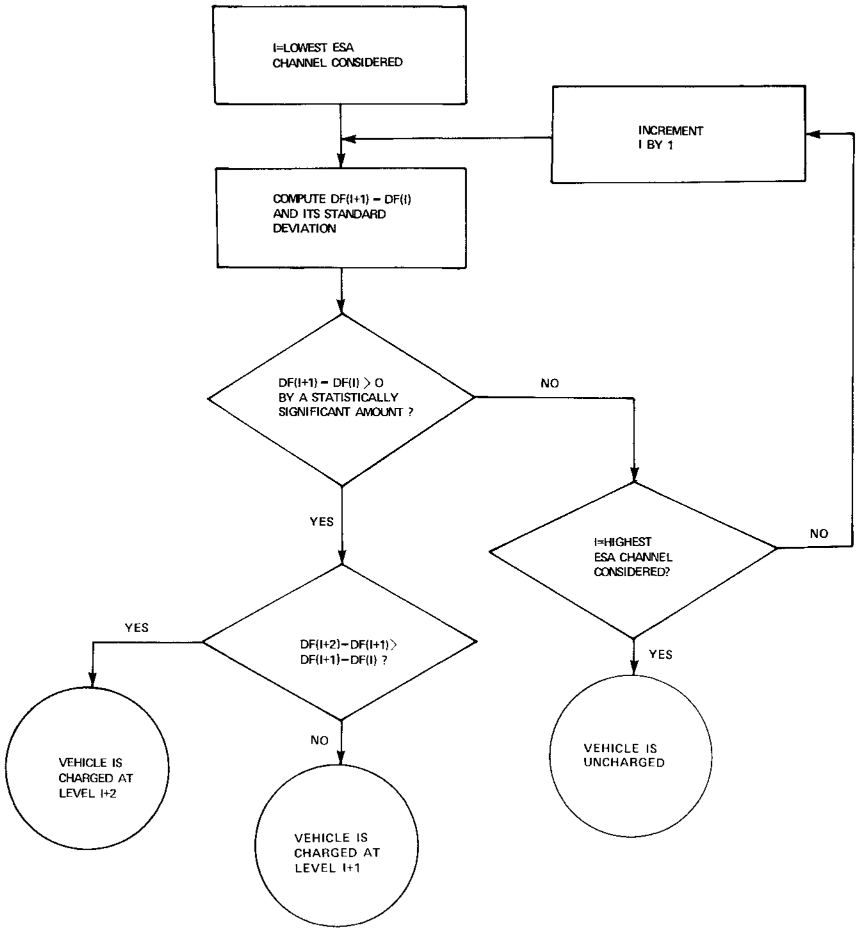

Fig. 1 Flowchart of the distribution function algorithm. The algorithm is based on comparisons of the ion distribution in adjacent energy channels [DF(I) and DF(I + 1)].

follows:

$$F(I) = \frac{1.70 \times 10^9 C(I)}{\left(1 + \dfrac{2}{1 + [E(I)/1500]^3}\right) E(I)^2} \qquad (2)$$

Assuming a Poisson distribution for the count rate, its standard deviation is $C(I)^{1/2}$ and its fractional standard deviation (FSD) is $C(I)^{-1/2}$. To determine the standard deviation of the denominator in Eq. (2), one must make an assumption concerning the precision of $E(I)$. A probable error of one or two percent is reasonable, hence an FSD of about 0.02 is assumed. This implies an FSD in $[E(I)]^2$ of about 0.04. For values of $E(I)$ in the region of interest for critical charging (about 1/2 kilovolt) the FSD of the remaining factor in the denominator is negligible. Thus the FSD in $F(I)$ is approximately

$$\mathrm{FSD}[F(I)] \approx \{(0.04)^2 + [C(I)^{-1/2}]^2\}^{1/2} \approx [0.002 + C(I)^{-1}]^{1/2} \qquad (3)$$

Finally, the standard deviation in $F(I+1) - F(I)$ is given by:

$$\mathrm{SD}[F(I+1) - F(I)] = \{[0.002 + C(I+1)^{-1}]F(I+1)^2 + [0.002 + C(I)^{-1}]F(I)^2\}^{1/2} \qquad (4)$$

The criterion for determining that the vehicle is charged is that

$$F(I+1) - F(I) \geq \gamma \mathrm{SD}[F(I+1) - F(I)] \qquad (5)$$

where γ, the standard deviation factor, is a positive constant. The choice $\gamma = 4$ has been found to give good results. In general, a high value of γ will reduce the fraction of the time the algorithm will falsely report charging (to less than $1/\gamma^2$ of the time by Chebyshev's theorem; see, for example, Freund and Walpole[5]), but this will be at the expense of increasing the number of times the algorithm will miss genuine charging. If the condition (5) is satisfied, the vehicle potential is taken to be at the level $E(I+1)$, unless it is found that $F(I+2) - F(I+1)$ exceeds $F(I+1) - F(I)$. In the latter case, the distribution

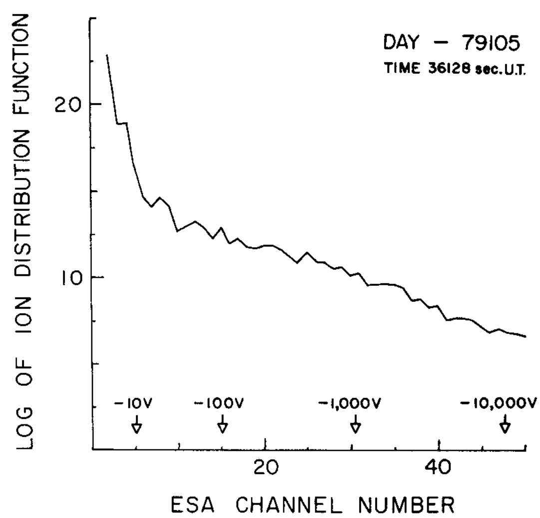

Fig. 2 Log of the positive ion particle distribution function as a function of SC9 high energy ESA channel for day 79105, time 36128 s UT. The distribution function exhibits the decrease with increasing energy expected for the case of an uncharged spacecraft.

Table 1 Performance of distribution function algorithm with SC9 data (in percent)

Day	Correct to within 20%	Correctly determines whether given critical potential is exceeded[a] 250 V	500 V
79086	93	97	98
79087	92	94	94
79098	100	100	99
79100	95	97	98
79104	99	100	100
79105	97	98	100
79106	95	100	100
79108	96	97	98
79114	86	97	99
79117	93	94	99
79118	93	99	99
79120	94	98	98
79172	98	100	100
79241	81	79	93
79267	91	95	99
79270	93	96	96
79271	88	95	96
79272	90	94	92
79273	89	92	93
79274	96	96	100
79276	93	96	97
79277	98	98	99
79282	99	100	100
79283	99	99	99
79285/6	100	100	99
79286	99	91	100
79294	100	100	100
79302	98	99	99
79305	99	99	100
80164	100	100	100
Overall	95.2	97.1	98.1

[a]In data base, VRS potential exceeded 250 V 29.4% of the time; VRS potential exceeded 500 V 23.6% of the time.

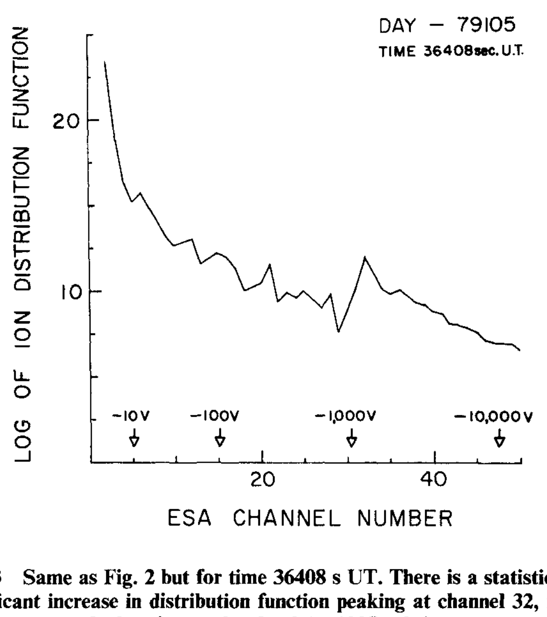

Fig. 3 Same as Fig. 2 but for time 36408 s UT. There is a statistically significant increase in distribution function peaking at channel 32, indicating spacecraft charging to that level (−1205 volts).

function profile suggests that the potential level be taken at $E(I+2)$ instead of $E(I+1)$, with the increase at channel $I+1$ attributable to the overlap in response characteristics of the ESA. A flowchart of the algorithm is shown in Fig. 1.

III. Tests of the Distribution Function Algorithm

The distribution function algorithm has been tested on the data base consisting of 9925 SC9 NS ion count spectra from time intervals on 30 different days containing known periods of vehicle charging used to test the count-ratio algorithm, described previously. As discussed in CR, informed estimates of the true vehicle potential were made for each of the spectra in the data base. These estimates were made by visual inspection of both high- and low-energy electron and ion count spectra from the SC9 instrument aboard P78-2. (The DC potential measurements of the SC10 experiment were not used because of their lack of reliability under eclipse conditions and at the high vehicle potentials that are associated with most of the charging events in our data base.) The algorithm's potential estimates were compared with these visually read spectrogram (VRS) potential estimates.

Typical ion distribution functions for day 79105 are shown in Fig. 2 (uncharged vehicle) and Fig. 3 (charged vehicle); for these cases, which are the same two that are shown in Figs. 3 and 4 of CR, the algorithm estimates are in agreement with the VRS potentials. It should be noted that the very large values of distribution function at low energies in Fig. 3 result from the small number of ion counts that appear in ESA channels below the level of charging, most likely due to instrument noise or possibly secondary ion production. The ESA characteristics are such that, in accordance with Eq. (1), a small number of counts leads to large values of the computed distribution function at low energies (tens of eV). However, the magnified increases in distribution function resulting from random count fluctuations at these low energies are not troublesome since they fail the algorithm's test for statistical significance.

A detailed comparison for the entire vehicle charging period on day 79105 is shown in Fig. 4. Results for the entire 30-day data base are given in the accompanying tables. The algorithm's performance is seen to have been quite impressive. Table 1 gives the day-by-day success rate in three categories:

1) Algorithm estimates vehicle potential correctly, to within 20% of VRS value.

2) Algorithm correctly decides if a critical potential of −250 V has been exceeded in magnitude.

3) Algorithm correctly decides if a critical potential of −500 V has been exceeded in magnitude.

The latter two categories are of great importance for the application of determining whether vehicle potential necessitates activating discharge mechanisms.

It is seen that, almost without exception, the performance is excellent. We intend to examine closely the data for a few of the days, most notably day 79241, to try to understand (and correct) the reasons for the occasional lower success rates. (It appears likely that in at least some instances it is the VRS potential estimates that are in need of occasional correction.) Comparison of these results with the corresponding numbers for the count-ratio algorithm (given in Table 1 of CR) shows that the distribution function algorithm is more successful in the large majority of cases (17 out of the 22 days for which the success rates of the two algorithms significantly differed).

Table 2 gives a summary of algorithm performance for the entire data base, comparing the distribution function and count-ratio algorithms. In addition to the categories of Table 1, Table 2 shows algorithm success in correctly determining the attainment of critical charge in two subcategories:

1) Algorithm makes correct decision when VRS potential exceeds critical.

2) Algorithm makes correct decision when VRS potential is less than critical.

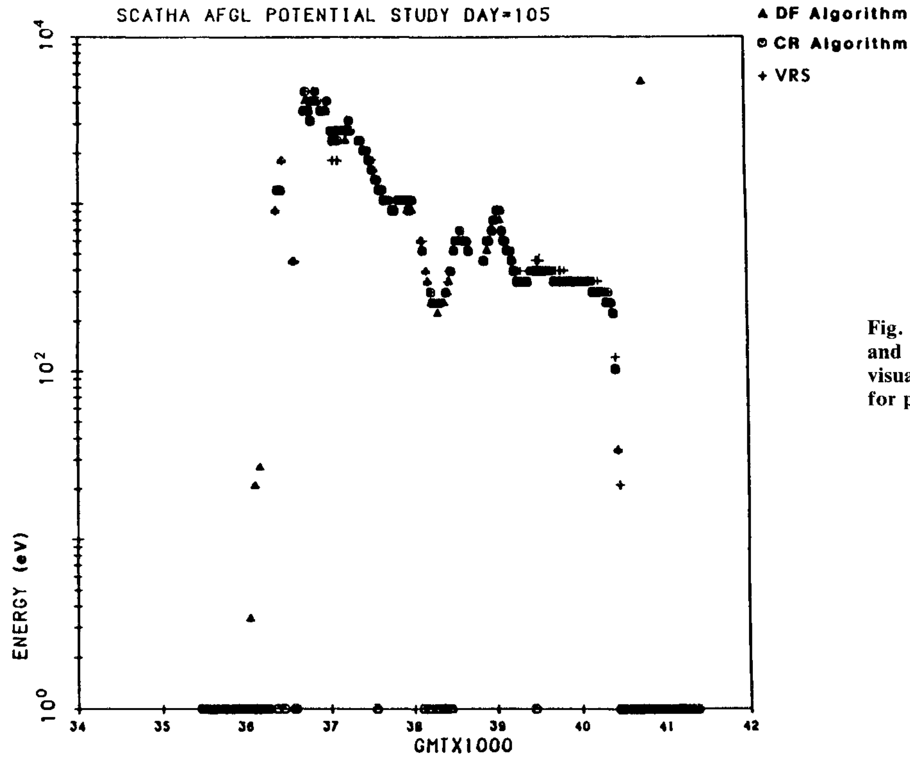

Fig. 4 Comparison of distribution function (DF) and count ratio (CR) algorithm estimates with visually read spectrogram (VRS) potential estimates for period of vehicle charging on day 79105.

The results for the count-ratio algorithm are reproduced from Table 3 of CR for convenience. The differences in success rates for the two algorithms in the various categories are all statistically significant at a confidence level exceeding 99.9%. It is seen that for the most part the distribution function algorithm is superior. The distribution function algorithm is substantially better at making the correct determination in cases where the critical potential has in fact been exceeded. The count-ratio algorithm has a slight edge in cases where critical charging does not take place; in fact it almost never gives a false alarm. Overall, however, the tests indicate that the distribution function algorithm is more reliable.

Additional testing has been performed, using data from the low resolution (8 channel) ESA of the SC5 experiment, and using SC9 data summed into 8 pseudochannels to simulate a lower resolution instrument (see CR for more details). These tests have only used count data from a few days of our 30-day data base but the results are quite similar to those for the count-ratio algorithm using all the data, reported in CR. The high overlap in response curves for adjacent channels of the SC5 ESA (Hanser et al.[6]) is believed to degrade algorithm performance with that instrument. Better results are obtained with the SC9 summed data (the effective response overlap is much smaller for these pseudochannels), although the success rates fail to equal those with the high resolution SC9 ESA data. Improved algorithm performance with this low resolution data awaits further analysis and testing.

Table 2 Comparison of distribution function and count ratio algorithm performance with SC9 (in percent)

	Distribution function algorithm	Count ratio algorithm
Correct to within 20%	95.2	92.0
Correct determination for 250 V critical potential	97.1	94.8
Correct when potential exceeds 250 V	93.4	82.6
Correct when potential less than 250 V	98.6	99.9
Correct determination for 500 V critical potential	98.1	96.5
Correct when potential exceeds 500 V	95.6	85.6
Correct when potential less than 500 V	98.9	99.9

IV. Conclusion

The results obtained thus far with the distribution function algorithm have been highly encouraging, especially with regard to the use of this algorithm with positive ion count data from an SC9 type ESA. It is particularly gratifying (and not entirely expected) that this algorithm worked so well without the imposition of any restriction that the test data be chosen from a stable plasma environment. Clearly, it has proven to be highly successful in the test intervals containing natural charging events irrespective of the plasma stability. The computational and storage requirements are suitable for real-time space vehicle potential determination using onboard microprocessors. The method works well and appears to be preferable to the count-ratio algorithm.

Further algorithm study and development are underway, in order, for example, to reduce the number of false reports of charging, and to make better determination of the optimal standard deviation factor γ. And, since the evaluation of the algorithm performance is dependent on how well the true satellite potential can be independently determined, methods of improving upon these estimates are being investigated.

The usefulness of this algorithm for SC5 data remains to be determined. It is possible that the charging detection procedure can be modified so that vehicle potential can be estimated with satisfactory accuracy even with considerable channel overlap and low energy resolution, despite the fact that distribution functions computed from these ESA data will frequently fail to reveal the increase with energy that occurs with vehicle charging. Instead, the computed distribution function may show a smaller decrease than that expected for an uncharged vehicle.

Indeed, tests in which the distribution function algorithm has been modified to search for a smaller-than-expected decrease (rather than an increase) in distribution function have shown improved results in a limited number of trials. With a better understanding of the ESA responses to a variety of ambient plasmas, it should be possible to find optimal criteria for charge detection with this type of instrument, and with ESA's of arbitrary design.

Acknowledgments

We wish to thank Drs. David A. Hardy and M. Susan Gussenhoven of Air Force Geophysics Laborabory, Space Physics Laboratory, for several helpful discussions. The first author wishes to acknowledge the support of the Air Force Office of Scientific Research/AFSC, United States Air Force, under Contract F49620-79-C-0038 and Grant AFOSR-85-0015. The United States government is authorized to reproduce and distribute reprints for governmental purposes notwithstanding any copyright notation herein.

References

[1]Spiegel, S. L., Saflekos, N. A., Gussenhoven, M. S., Raistrick, R. J., and Cohen, H. A., "Real-Time, Automatic Vehicle-Potential Determination from ESA Measurements: The Count-Ratio Algorithm," *Journal of Spacecraft and Rockets*, Vol. 25, Jan.–Feb. 1988, pp. 59–63.

[2]Stevens, J. R. and Vampola, A. L. (eds.), "Description of the Space Test Program P78-2 Spacecraft and Payloads," United States Air Force Space Division, Los Angeles, CA, SAMSO TR-78-24, Oct. 1978, pp. 49–53.

[3]Mullen, E. G., Garrett, H. B., Hardy, D. A., and Whipple, E. C., "P78-2 SCATHA Preliminary Data Atlas," Air Force Geophysics Laboratory, Hanscom Air Force Base, MA, AFGL-TR-80-0241, Aug. 1980, pp. 42–43.

[4]de Forest, S. D., "Spacecraft Charging at Synchronous Orbit," *Journal of Geophysical Research*, Vol. 77, No. 4, pp. 651–659.

[5]Freund, J. E. and Walpole, R. E., *Mathematical Statistics*, 3rd ed., Prentice-Hall, Englewood Cliffs, NJ, 1980, p. 54.

[6]Hanser, F. A., Hardy, D. A., and Sellers, B., "Calibration of the Rapid Scan Particle Detector Mounted in the SCATHA Satellite," Air Force Geophysics Laboratory, Hanscom Air Force Base, MA, AFGL-TR-79-0167, 1979, pp. 16–22.